THE WHAT, WHY, AND HOW OF NUMBER BONDS AND BAR MODELS

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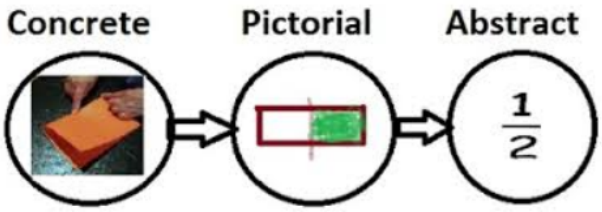
Our approach for teaching mathematics in the Wappingers Central School District is by using the concrete-pictorial-abstract approach. Both our textbook series "Math in Focus" and the EngageNY Modules follow this approach. Concepts are introduced using something concrete- a manipulative or item that students can touch, move, fold, etc. From there, they transition into a pictorial approach in which a drawing or diagram is used to represent the concept. Finally, students move to the abstract- using algorithms and operations to solve.



This approach is beneficial to students because it appeals to various learning styles. It also develops and understanding of "why" a mathematical algorithm works, not just "how" to do it.

At our Parent Workshop Series on April 2, 2016, we were able to discuss the pictorial representations that are most frequently used in grades K-6 in depth. While the slides from this session are attached to this document, we have also prepared three quick videos that you can view in case you were unable to attend the presentation or if you would like to revisit any of the content.

Brief Overview of Pictorial Models: https://goo.gl/7mhsnO

Brief Overview of Fraction Models: https://goo.gl/r8TRzJ

Different ways of adding: Why we want to build our "toolkit": https://goo.gl/8DdnG2

Thank you for joining us!

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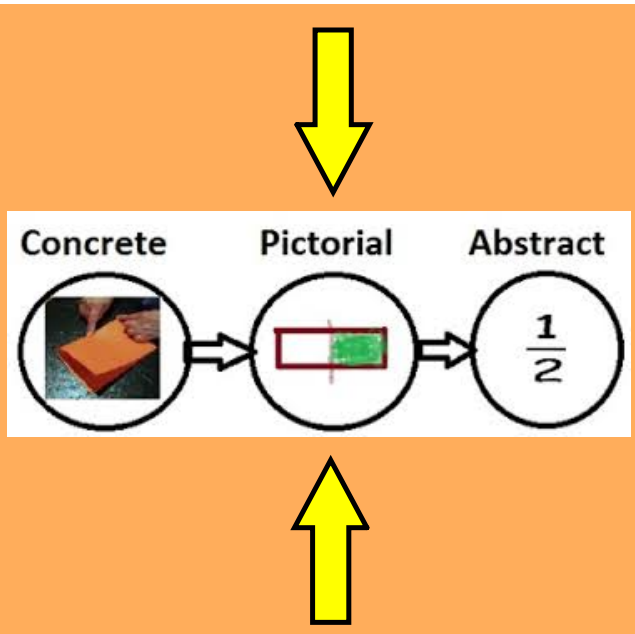
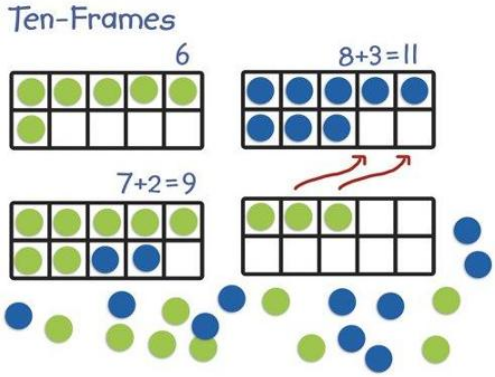
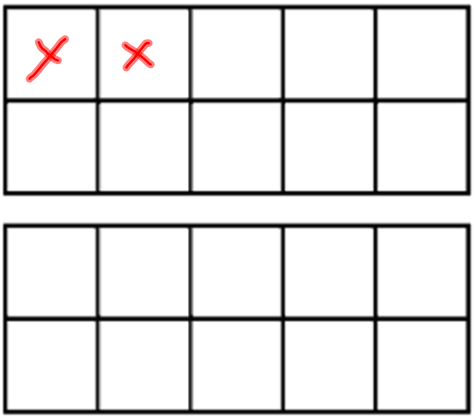

Concrete
Pictorial
Abstract
1
2

TEN FRAME

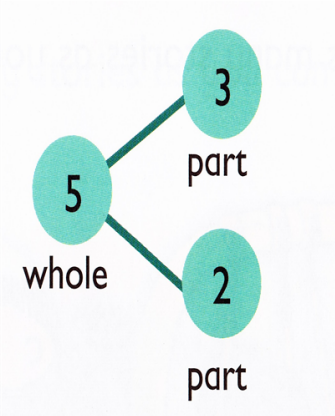
Concrete

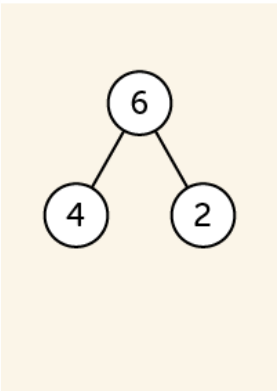


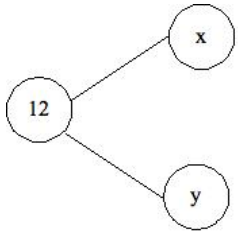
Pictorial



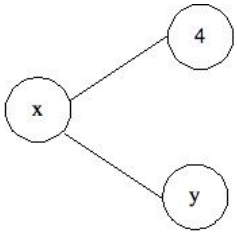
NUMBER BOND





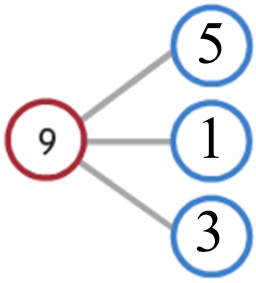


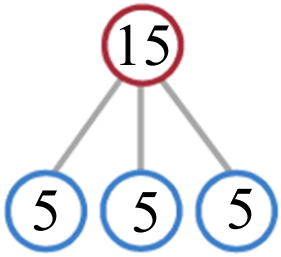
x+y=12



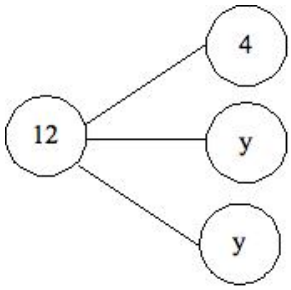
x-y=4

NUMBER BONDS (continued)



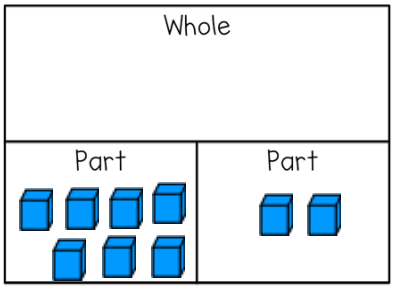


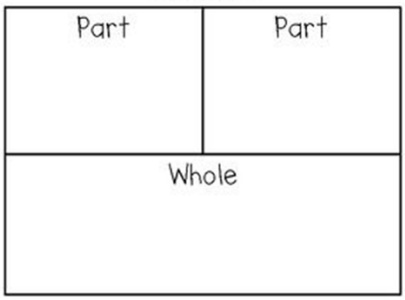
5 + 5 + 5 = 15
or
5 x 3 = 15

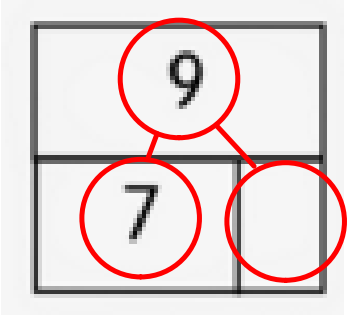


2y + 4 = 12

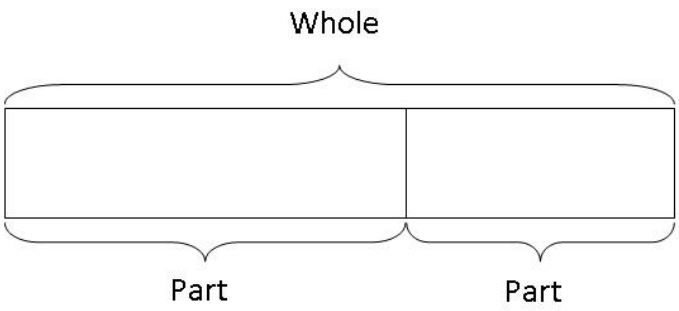
PART-PART-WHOLE





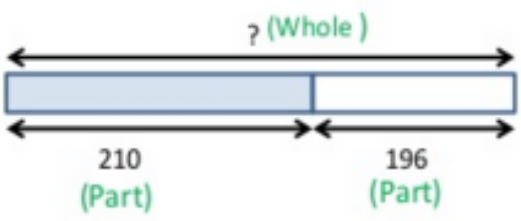


ADDITION and SUBTRACTION BAR MODEL (aka TAPE DIAGRAM)



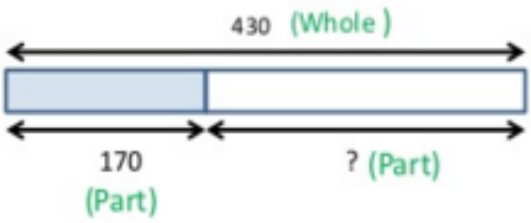
a) To find whole:

- part + part = whole



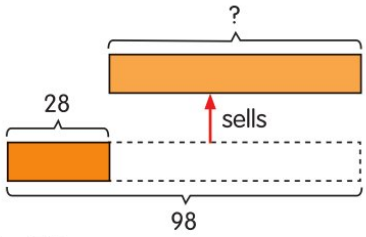
b) To find part:

- whole – part = part



Another way of seeing subtraction....

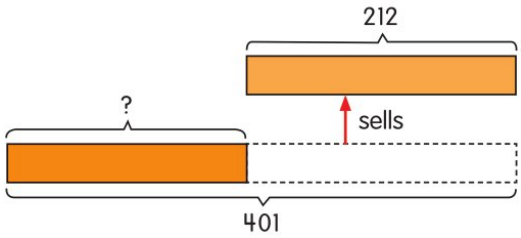
A florist has 98 flowers.
She sells some of them.
She has 28 flowers left.
How many flowers does she sell?



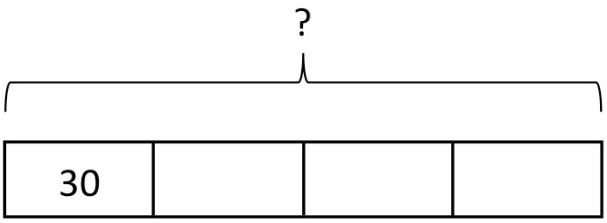
98 – 28 = 70

She sells 70 flowers.

Lena has 401 trading cards in her shop.
She sells 212 trading cards.
How many trading cards
does she have left?



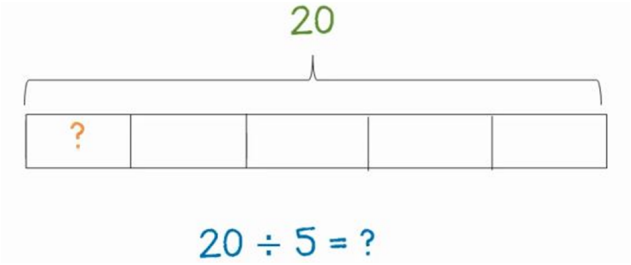
MULTIPLICATION BAR MODEL (aka TAPE DIAGRAM)



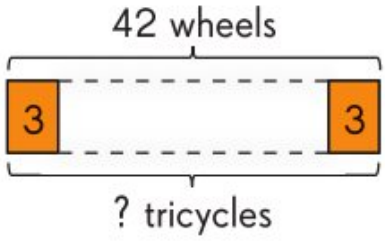
$30 \times 4 = 120$

DIVISION BAR MODEL
(aka TAPE DIAGRAM)

Known number of groups.

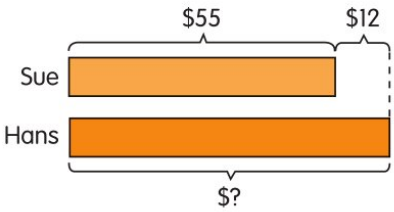


How many in each group is known.



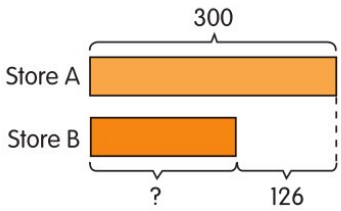
COMPARISON BAR MODEL (aka TAPE DIAGRAM)

Sue has $55.
Hans has $12 more than Sue.
How much money does Hans have?



55 + 12 = ?

In Store A, 300 video games are sold.
This is 126 more games sold than in Store B.
How many video games are sold in Store B?

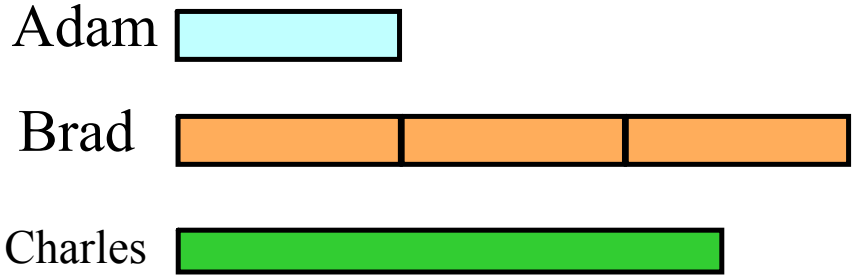


300 - 126 = ?


A
B
B - A
A + B

COMPARISON BAR MODEL (aka TAPE DIAGRAM)

Can be used for multiple comparisons.



Brad has 3 times the amount of cards as Adam does. Charles has 9 less cards than Brad. Adam has 11 cards. How many do they have in total?

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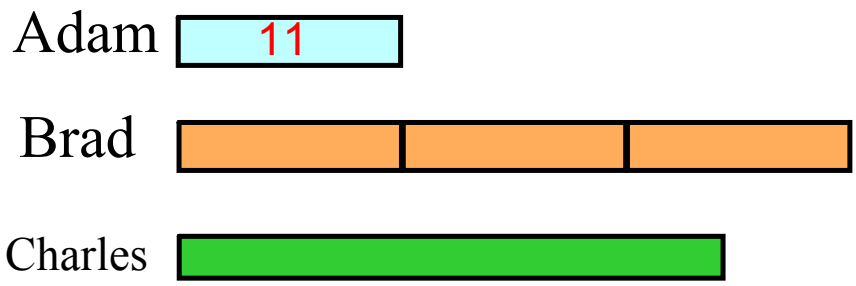

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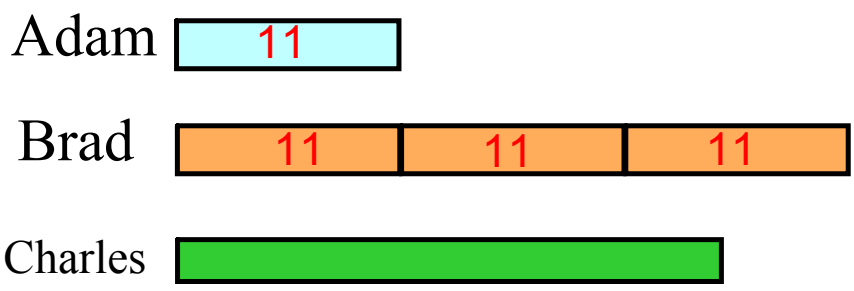

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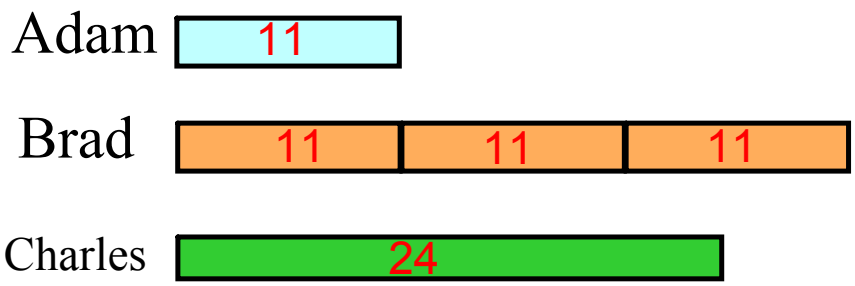

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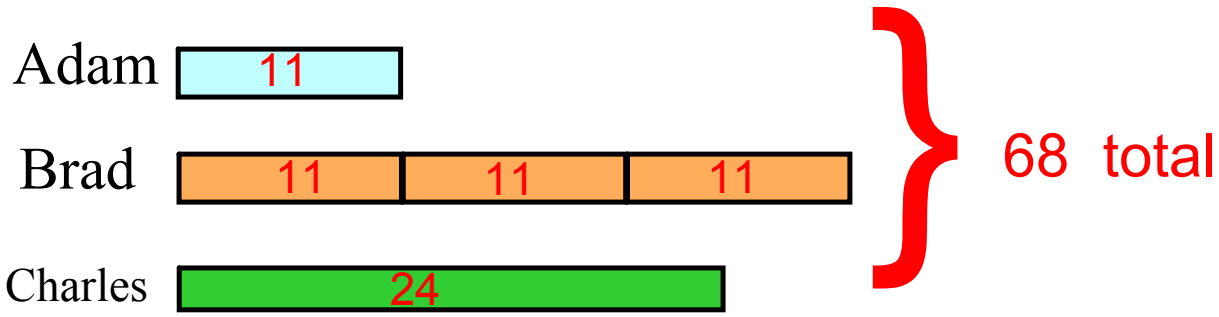
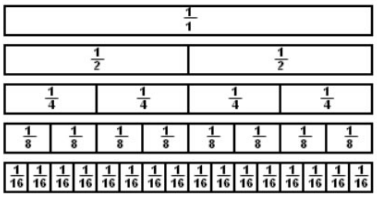
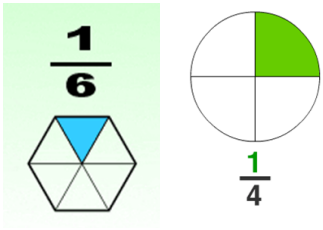
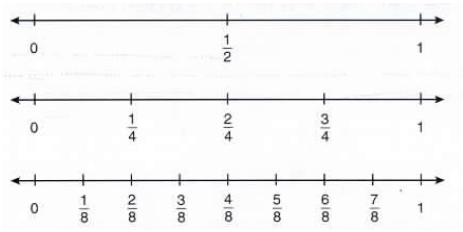
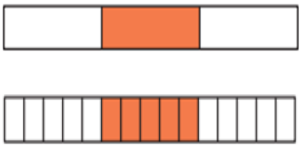
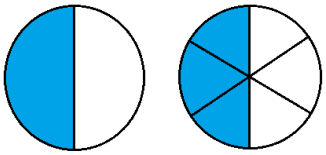
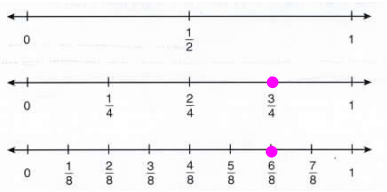


Using Models with Fractions

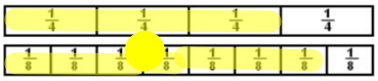
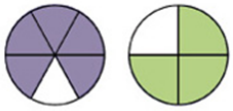
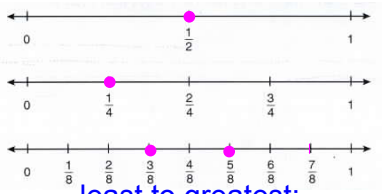
UNIT FRACTION

Bar	Figure	Number Line
		

FRACTION EQUIVALENCY

Bar	Figure	Number Line
 $\frac{1}{3}$ $\frac{5}{15}$	 1/2 3/6	

COMPARE/ORDER FRACTIONS

Bar	Figure	Number Line
		
$\frac{3}{4} < \frac{7}{8}$	$\frac{5}{6} > \frac{3}{4}$	least to greatest: $\frac{1}{4}$ $\frac{3}{8}$ $\frac{1}{2}$ $\frac{5}{8}$ $\frac{7}{8}$

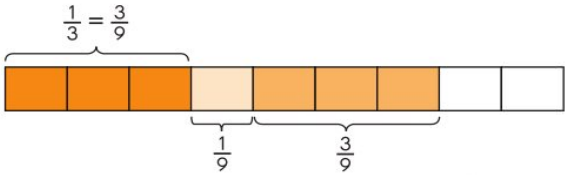
ADDING/SUBTRACTING FRACTIONS (can use any figure or a number line)

Three friends shared a grapefruit.

Elena ate $\frac{1}{3}$ of the grapefruit.

Lee ate $\frac{1}{9}$ of the grapefruit.

Sara ate $\frac{3}{9}$ of the grapefruit.



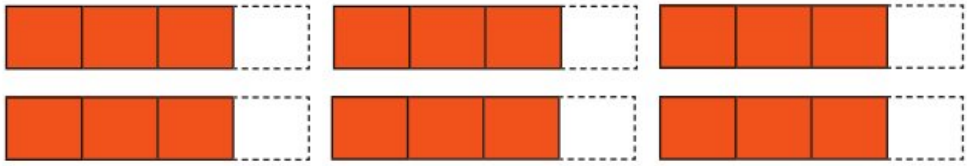
What fraction of the grapefruit did they eat altogether?

$$\frac{1}{3} + \frac{1}{9} + \frac{3}{9} = \frac{3}{9} + \frac{1}{9} + \frac{3}{9}$$
$$= \frac{7}{9}$$

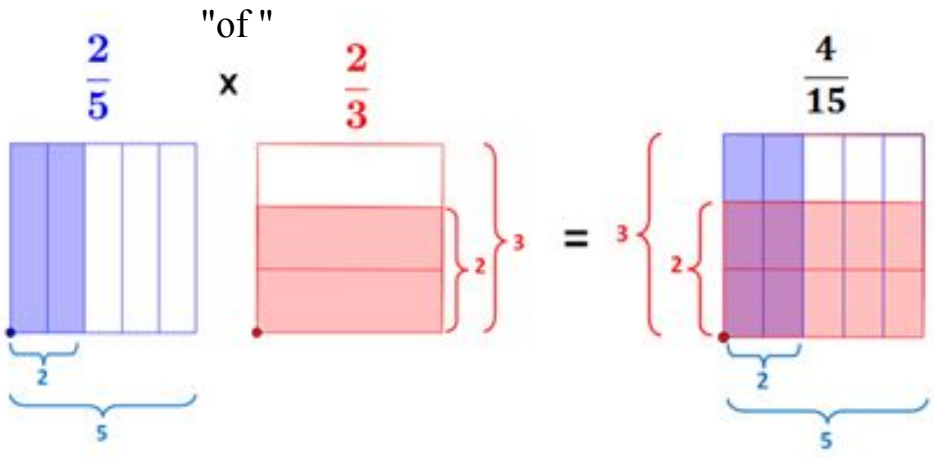
Elena, Lee, and Sara ate $\frac{7}{9}$ of the grapefruit.

FRACTION MULTIPLICATION
(can use any type of figure)

Find $6 \times \frac{3}{4}$.

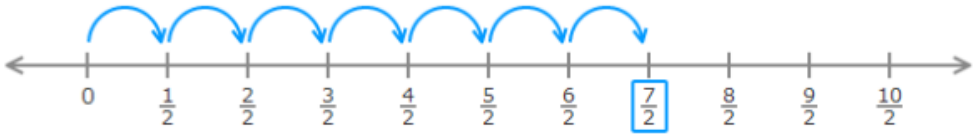


FRACTION MULTIPLICATION: Area Model



FRACTION MULTIPLICATION: Number Line

The model shows the product of 7 and $\frac{1}{2}$.



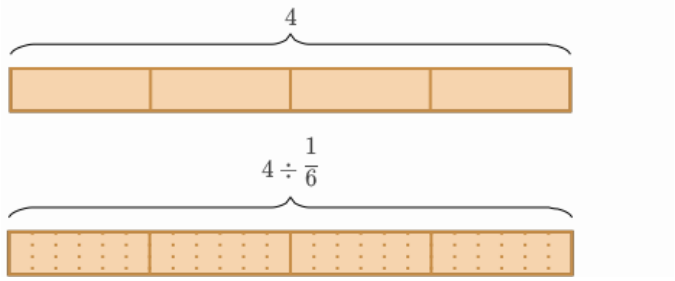
Complete the multiplication sentence that matches the model.

$7 \times \frac{1}{2} = \frac{\square}{\square}$

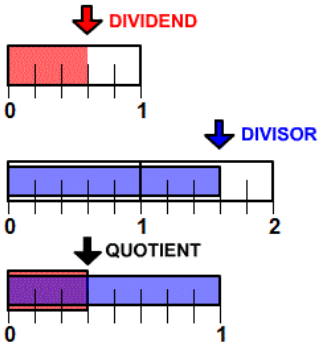
FRACTIONS AND DIVISION

If 4 wholes are divided into pieces that are each $\frac{1}{6}$ of a whole, how many pieces are there?

$4 \div \frac{1}{6} = \square$



FRACTIONS AND DIVISION



$\frac{3}{5} \div 1\frac{3}{5}$

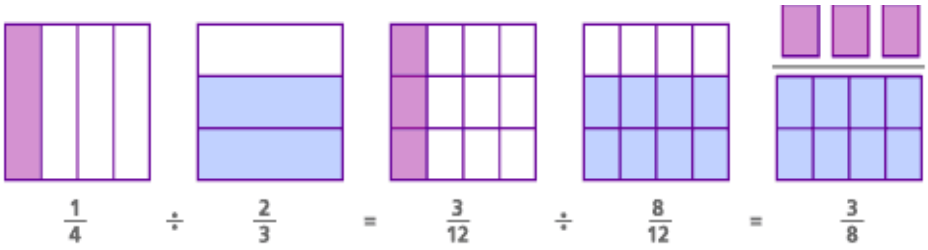
dividend divisor

$\frac{3}{5} \div 1\frac{3}{5} = \frac{3}{5} \div \frac{8}{5}$ Write in fraction form.

$= \frac{3}{5} \times \frac{5}{8} = \frac{15}{40}$ Multiply by the reciprocal

$= \frac{3}{8}$ simplify

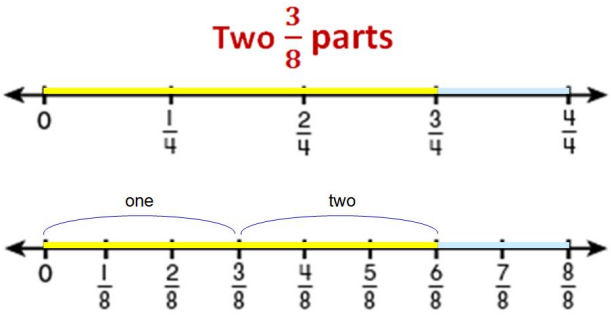
FRACTIONS AND DIVISION:
Common Denominator Area Model



DIVIDING FRACTIONS: NUMBER LINE

$\frac{3}{4} \div \frac{3}{8} =$ 2

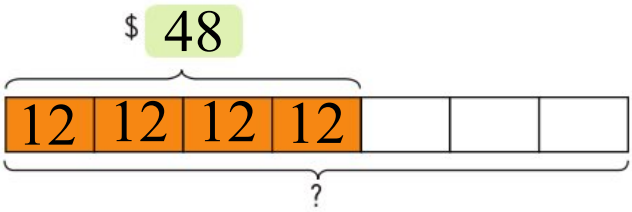
How many $\frac{3}{8}$ parts can be partitioned from $\frac{3}{4}$?



FRACTIONS OF A SET

What is $\frac{2}{7}$ of 49?

Vincent spent $\frac{4}{7}$ of his money on a pair of shoes.
The shoes cost $48. How much money did he have at first?



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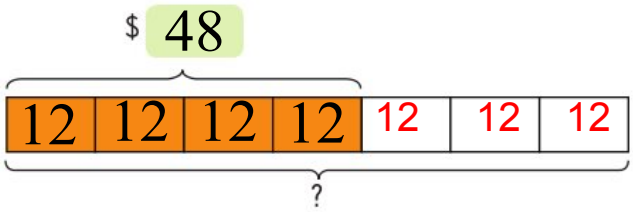
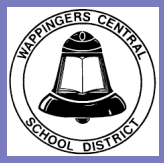


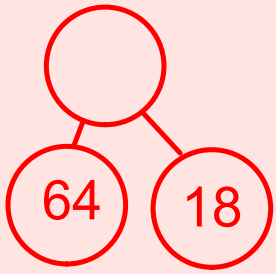
12 x 7
$84

"The Toolkit"

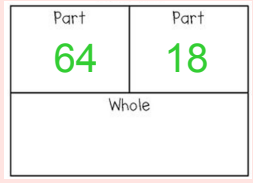


What is 64 + 18?

Number Bond



Part-Part-Whole Mat



Bar Model

How will we find the sum?

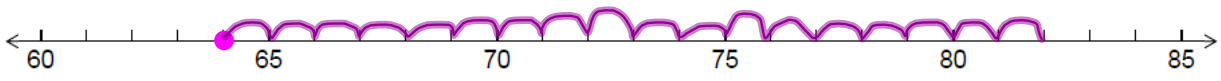
What is 64 + 18?

Counting on:
65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82

Counting on with 10:
"18 is equal to a 10 plus an 8. So 10 more than 64 is 74, then 75, 76, 77, 78, 79, 80, 81, 82"

What is 64 + 18?

Using a number line:



What is 64 + 18?

Using a benchmark value:

18 is 2 fewer than 20.
So 64 + 20 = 84, and 2 fewer than 84 is 82.
So 64 + 18 = 82

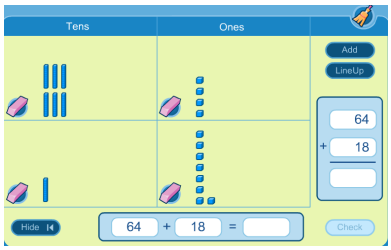
What is 64 + 18?

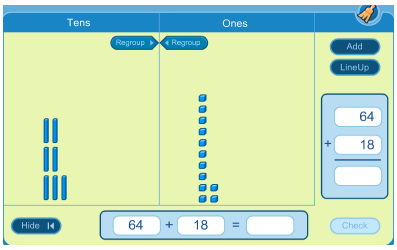
Partial Sums:

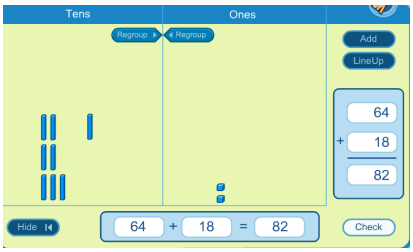
"64 is 6 tens and 4 ones.
18 is 1 ten and 8 ones.
So I have 7 tens and 12 ones.
70 + 12
equals 82"

What is 64 + 18?

Base 10 Blocks:







https://www-k6.thinkcentral.com/content/hsp/math/mathinfocus/common/itools_pri_9780547673851_/basetenblocks.html

Decomposition:

"64 is 6 away from 70. So if I take 6 from the 18, I am left with 12. So I can do 70 + 12 = 82."

What is 64 + 18?

Standard Algorithm:

$$\begin{array}{r} {\scriptstyle 1} \\ 64 \\ +\,18 \\ \hline 82 \end{array}$$

We expose our students to a variety of methods so that they are able to expand their personal "toolkit." When it comes to execution, they can choose which "tool" to use.